# Supplementary Material

# A new nonparametric test for the race model inequality

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**Abstract**

In this document, we first report the full version of the results of the second Monte Carlo simulation study which were presented in a more compact way in the main text of the manuscript (see Section 3.2). Furthermore, here we also show the results of an additional power analysis study (not reported in the manuscript) based on the Diffusion Superposition Model (Schwarz, 1994) which is a well known coactivation model for reaction times. In both the simulation studies we contrasted the truncated KS test against its mixture counterpart originally proposed by Maris and Maris (2003).

# 1 Power analysis simulation studies

Section 1.1 shows the complete Monte Carlo simulation study conducted within the Weibull distributional framework which was concisely described in Section 3.2 of our manuscript. Section 1.2 instead reports the results of an additional Monte Carlo simulation study not presented in the manuscript which is based on the Diffusion Superposition Model (Schwarz, 1994).

## 1.1 Power analysis simulation study 1

The detailed description of the factorial design of this first simulation study is reported in Section 3.2 of our original manuscript. The following tables (Tables 1-4) and figures (Figures 1-4) show rejection rates of the null hypothesis (separately for the truncated Kolmogorov-Smirnov test and the mixture Kolmogorov-Smirnov test) for each of the four blocks ($\delta = 0$, $\delta = 0.1$ $\delta = 0.2$ and $\delta = 0.3$) in the complete Monte Carlo simulation design.

Table 1: Power analysis (as percentage of rejections) for the truncated KS test (*first row*), and the mixture KS test (*second row*), for $\delta = 0$. MG: median gain; MTD: maximal theoretical distance.

| | *MG* | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| sample size ($s$) | -5.70 | -2.70 | -0.28 | 2.34 | 4.20 | 6.16 | 7.84 | 9.42 | 10.83 | 12.15 |
| 20 | 0.8 | 1.4 | 2.3 | 3.5 | 5.0 | 7.1 | 10.0 | 12.8 | 17.7 | 20.9 |
| | 1.7 | 2.8 | 4.5 | 5.1 | 7.3 | 9.9 | 12.7 | 15.4 | 17.3 | 21.5 |
| 50 | 0.4 | 0.9 | 1.9 | 3.2 | 6.3 | 10.3 | 15.4 | 21.6 | 27.2 | 37.7 |
| | 0.9 | 1.8 | 3.3 | 4.9 | 8.9 | 11.1 | 15.2 | 20.8 | 27.3 | 34.1 |
| 100 | 0.0 | 0.6 | 1.5 | 3.7 | 9.1 | 15.2 | 24.2 | 36.7 | 49.2 | 60.7 |
| | 0.6 | 1.8 | 3.5 | 7.2 | 11.1 | 18.4 | 25.9 | 36.0 | 46.4 | 56.9 |
| 250 | 0.1 | 0.4 | 1.6 | 6.5 | 17.4 | 35.5 | 55.9 | 74.9 | 88.7 | 95.5 |
| | 0.2 | 1.1 | 2.8 | 8.8 | 18.8 | 33.3 | 49.1 | 65.4 | 78.7 | 89.0 |
| 500 | 0.0 | 0.4 | 3.3 | 13.8 | 39.3 | 69.5 | 88.9 | 97.7 | 99.7 | 99.9 |
| | 0.1 | 0.9 | 4.0 | 13.2 | 33.8 | 57.0 | 78.6 | 91.8 | 97.1 | 99.3 |
| *MTD* | 0 | 0.01 | 0.02 | 0.04 | 0.07 | 0.09 | 0.11 | 0.14 | 0.16 | 0.18 |

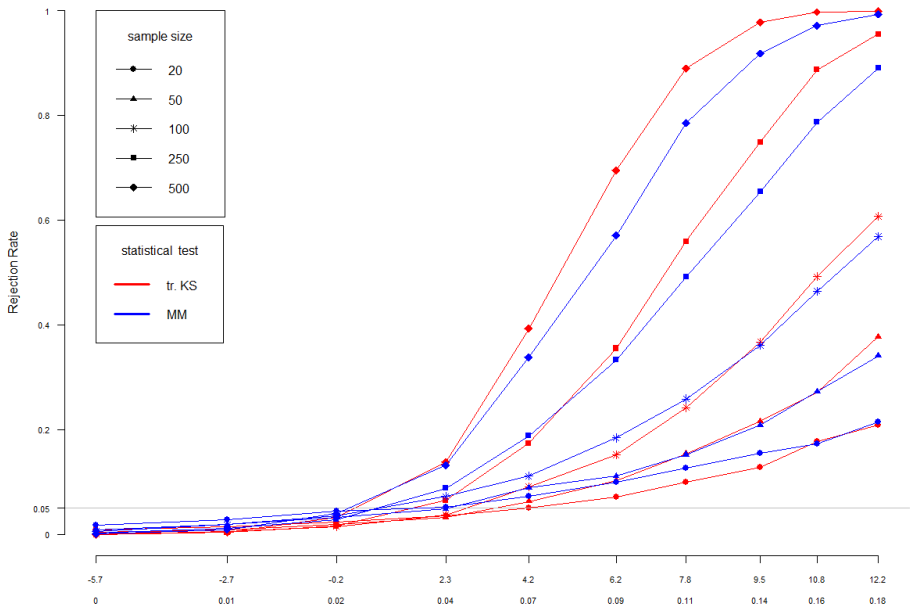


Figure 1: Power analysis as a function of sample size and distance information (MG: first row, MTD: second row) for block $\delta = 0$. Black lines denote the truncated KS test. Blue lines indicate the mixture KS test.

Table 2: Power analysis (as percentage of rejections) for the truncated KS test (*first row*), and the mixture KS test (*second row*), for $\delta = 0.1$. MG: median gain; MTD: maximal theoretical distance.

| | *MG* | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| sample size ($s$) | -5.60 | -2.24 | 0.76 | 3.36 | 5.60 | 7.66 | 9.56 | 11.31 | 12.89 | 14.29 |
| 20 | 0.7 | 1.8 | 2.6 | 5.1 | 7.3 | 9.8 | 13.1 | 17.7 | 21.5 | 28.4 |
| | 2.1 | 2.9 | 5.0 | 6.3 | 9.3 | 12.3 | 15.4 | 19.4 | 22.2 | 26.7 |
| 50 | 0.5 | 0.9 | 2.3 | 4.9 | 8.4 | 14.4 | 21.0 | 29.2 | 41.3 | 49.9 |
| | 1.1 | 2.1 | 3.5 | 6.6 | 10.6 | 15.7 | 21.9 | 28.3 | 36.8 | 43.6 |
| 100 | 0.2 | 0.5 | 1.6 | 4.9 | 10.9 | 22.2 | 35.1 | 49.8 | 65.1 | 78.1 |
| | 0.3 | 1.4 | 3.1 | 6.6 | 12.6 | 20.1 | 32.4 | 44.1 | 55.3 | 65.6 |
| 250 | 0.0 | 0.4 | 2.2 | 9.9 | 26.6 | 50.6 | 74.3 | 88.7 | 96.1 | 99.1 |
| | 0.2 | 0.7 | 3.6 | 11.8 | 25.7 | 44.0 | 64.0 | 80.8 | 91.2 | 96.5 |
| 500 | 0.0 | 0.2 | 3.4 | 18.9 | 54.2 | 84.1 | 96.7 | 99.7 | 99.9 | 100 |
| | 0.0 | 0.8 | 5.1 | 18.7 | 46.0 | 72.5 | 90.7 | 97.9 | 99.7 | 100 |
| *MTD* | 0 | 0.00 | 0.02 | 0.05 | 0.08 | 0.11 | 0.14 | 0.16 | 0.19 | 0.22 |

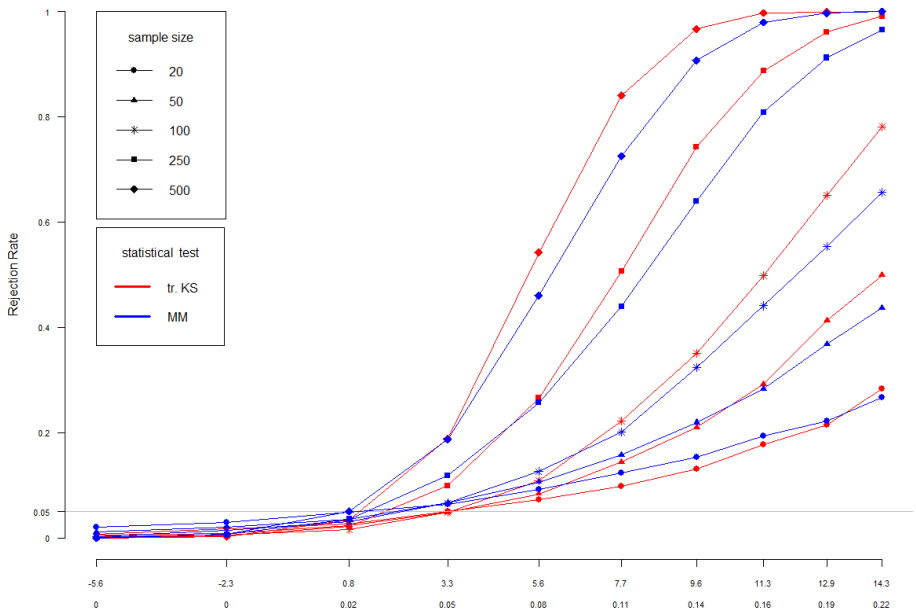


Figure 2: Power analysis as a function of sample size and distance information (MG: first row, MTD: second row) for block $\delta = 0.1$. Black lines denote the truncated KS test. Blue lines indicate the mixture KS test.

Table 3: Power analysis (as percentage of rejections) for the truncated KS test (*first row*), and the mixture KS test (*second row*), for $\delta = 0.2$. MG: median gain; MTD: maximal theoretical distance.

| | *MG* | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| sample size ($s$) | -5.88 | -2.24 | 1.12 | 3.92 | 6.44 | 8.97 | 10.93 | 12.89 | 14.57 | 16.25 |
| 20 | 1.2 | 2.0 | 3.2 | 5.3 | 7.4 | 12.2 | 16.5 | 21.1 | 27.0 | 32.3 |
| | 2.0 | 3.1 | 4.8 | 7.6 | 10.2 | 14.5 | 18.4 | 23.2 | 27.2 | 33.3 |
| 50 | 0.5 | 0.9 | 2.6 | 5.8 | 10.7 | 17.8 | 27.7 | 39.3 | 51.3 | 62.7 |
| | 0.7 | 1.9 | 4.3 | 7.3 | 13.0 | 19.1 | 26.5 | 35.1 | 44.6 | 52.7 |
| 100 | 0.1 | 0.7 | 2.0 | 6.5 | 15.6 | 29.8 | 47.8 | 65.4 | 79.0 | 88.7 |
| | 0.5 | 1.7 | 4.6 | 10.2 | 18.9 | 30.2 | 44.5 | 58.5 | 70.5 | 79.9 |
| 250 | 0.0 | 0.3 | 2.4 | 12.6 | 35.6 | 65.1 | 85.9 | 95.9 | 99.4 | 99.9 |
| | 0.0 | 0.1 | 4.4 | 14.6 | 31.8 | 56.2 | 75.8 | 90.2 | 96.1 | 98.4 |
| 500 | 0.0 | 0.1 | 3.6 | 24.6 | 65.3 | 93.0 | 99.3 | 100 | 100 | 100 |
| | 0.0 | 0.7 | 6.0 | 25.3 | 58.5 | 83.8 | 96.6 | 66.5 | 99.9 | 100 |
| *MTD* | 0 | 0.00 | 0.02 | 0.05 | 0.09 | 0.12 | 0.16 | 0.19 | 0.22 | 0.25 |

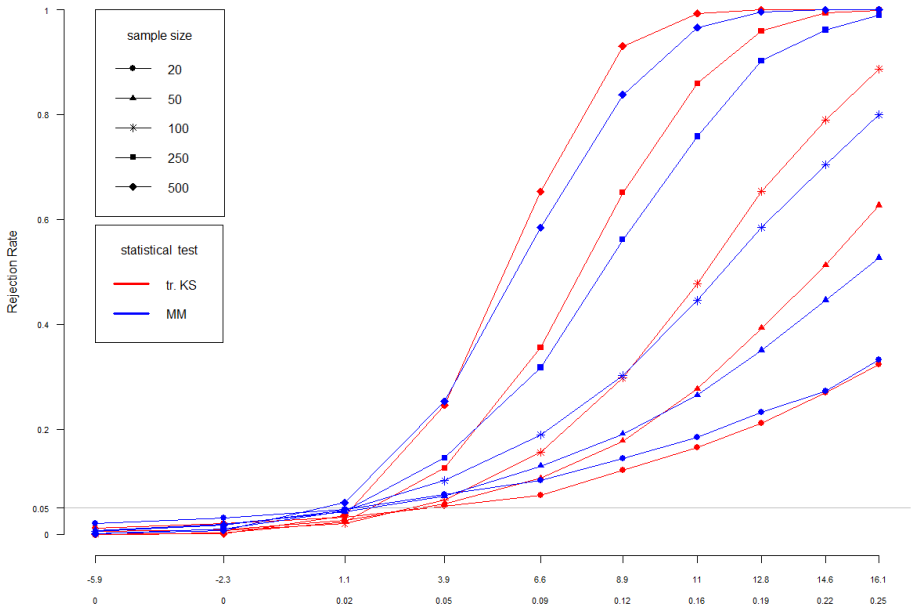


Figure 3: Power analysis as a function of sample size and distance information (MG: first row, MTD: second row) for block $\delta = 0.2$. Black lines denote the truncated KS test. Blue lines indicate the mixture KS test.

Table 4: Power analysis (as percentage of rejections) for the truncated KS test (*first row*), and the mixture KS test (*second row*), for $\delta = 0.3$. MG: median gain; MTD: maximal theoretical distance.

| | *MG* | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| sample size ($s$) | -6.89 | -2.56 | 1.12 | 4.40 | 7.29 | 9.81 | 12.05 | 14.01 | 15.98 | 17.66 |
| 20 | 0.7 | 1.7 | 3.1 | 6.3 | 8.7 | 13.3 | 18.8 | 25.3 | 31.4 | 38.9 |
| | 2.0 | 4.0 | 6.5 | 8.5 | 12.2 | 17.0 | 21.0 | 26.5 | 31.8 | 36.1 |
| 50 | 0.3 | 0.8 | 2.5 | 6.0 | 11.5 | 20.7 | 32.7 | 43.4 | 58.3 | 70.7 |
| | 0.7 | 1.9 | 4.4 | 8.5 | 14.1 | 22.5 | 31.4 | 42.1 | 52.5 | 61.3 |
| 100 | 0.0 | 0.4 | 2.1 | 7.9 | 18.4 | 36.5 | 59.1 | 75.8 | 87.1 | 94.7 |
| | 0.2 | 1.2 | 3.5 | 10.7 | 20.3 | 32.9 | 47.7 | 63.5 | 75.1 | 86.1 |
| 250 | 0.0 | 0.2 | 2.3 | 14.4 | 43.1 | 75.0 | 93.9 | 98.9 | 99.9 | 100 |
| | 0.1 | 0.8 | 4.4 | 17.5 | 39.9 | 65.0 | 85.3 | 94.5 | 98.5 | 99.8 |
| 500 | 0.0 | 0.0 | 3.5 | 27.7 | 76.1 | 97.3 | 99.9 | 100 | 100 | 100 |
| | 0.0 | 0.3 | 4.8 | 26.2 | 63.5 | 89.6 | 98.2 | 99.9 | 100 | 100 |
| *MTD* | 0 | 0.00 | 0.01 | 0.06 | 0.10 | 0.14 | 0.18 | 0.21 | 0.25 | 0.28 |

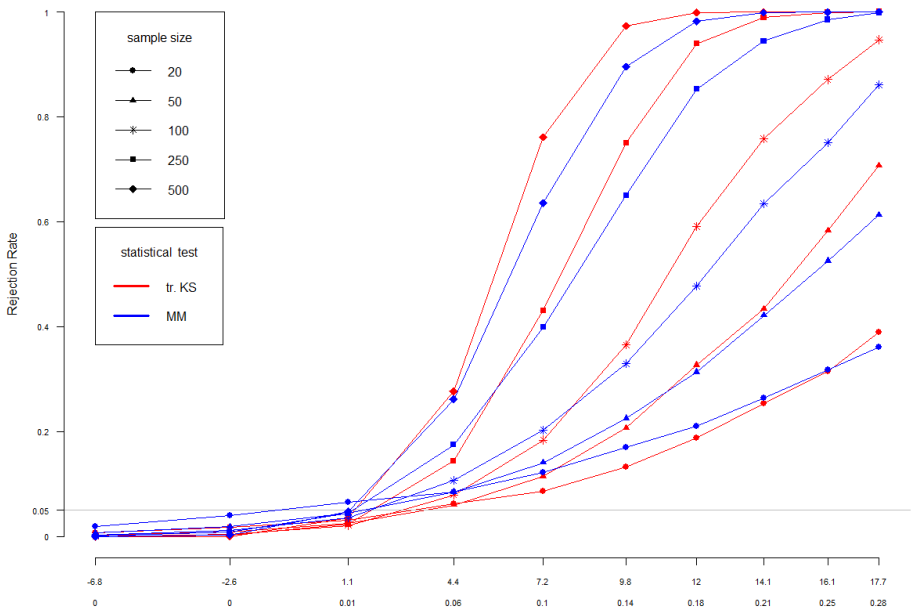


Figure 4: Power analysis as a function of sample size and distance information (MG: first row, MTD: second row) for block $\delta = 0.3$. Black lines denote the truncated KS test. Blue lines indicate the mixture KS test.

## 1.2 Power analysis simulation study 2

In order to assess the power of the truncated Kolmogorov-Smirnov test an additional simulation study was performed using the Diffusion Superposition Model framework (Schwarz, 1994). This new simulation study was based on a design with two factors: 1) a dichotomous (0/1) factor denoting the theoretical violation of the race model (coded as 1) 2) the sample size factor at five levels: 20, 50, 100, 250, and 500. Reaction times were simulated according to the Diffusion Superposition Model (Schwarz, 1994) for both single signals and redundant signals conditions. As usual, let $X$ and $Y$ represent the channels for the two single signals trials. Reaction times were sampled from an *inverse Gaussian* distribution ($IG$). To set the values of the model parameters we took advantage of the simulation design reported in Gondan (2010). In particular, we sampled the simulated reaction times according to the following distributions: $X \sim IG(a = 100, \mu_X = 1.34, \sigma_X = 11.7)$ and $Y \sim IG(a = 100, \mu_Y = 0.53, \sigma_Y = 4.3)$, where $a$ is the position of the absorbing barrier, and $\mu$ and $\sigma$ are the diffusion parameters (resp. drift and standard deviation). Under the race model assumption, reaction times for the redundant signals condition were generated according to the maximal statistical facilitation attainable by a race model. The latter required to use the specific equations for the reconstructed distribution $K$ derived under the truncated (resp. the mixture) representation. By contrast, in order to generate observations that violated the race model, the reaction times were directly simulated according to the coactivation model. In particular, $Z \sim IG(a = 100, \mu_Z = \mu_X + \mu_Y, \sigma_Z^2 = \sigma_X^2 + \sigma_Y^2)$. Results are shown in Table 5.

Table 5: Power analysis (as percentage of rejections) for the truncated KS test and mixture KS test (in parenthesis)

| | *sample size* | | | | |
|---|---|---|---|---|---|
| | 20 | 50 | 100 | 250 | 500 |
| Race Model | 2.6 (8.5) | 1.2 (6.4) | 2.1 (5.6) | 2.0 (6.6) | 2.1 (6.3) |
| Coactivation Model | 8.8 (13.6) | 14.9 (17.7) | 24.8 (26.9) | 66.0 (52.9) | 92.6 (82.2) |

We further considered a second simulation study with different parameters assignment for the superposition model. In the new simulation we set $X \sim IG(a = 100, \mu_X = 1.54, \sigma_X = 8.7)$ and $Y \sim IG(a = 100, \mu_Y = 1.25, \sigma_Y = 6.5)$. Results are reported in Table 6.

Table 6: Power analysis (as percentage of rejections) for the truncated Kolmogorov-Smirnov test and Maris & Maris test (in parenthesis)

| | *sample size* | | | | |
|---|---|---|---|---|---|
| | 20 | 50 | 100 | 250 | 500 |
| Race Model | 5.7 (9.4) | 6.7 (7.3) | 5.1 (7.3) | 5.2 (7.5) | 5.4 (7.7) |
| Coactivation Model | 32.2 (28.1) | 63.7 (53.4) | 92.8 (81.7) | 99.9 (99.7) | 100 (100) |

## 1.3 Final comments

Overall, the truncated Kolmogorov-Smirnov test seemed to perform better than the statistical test proposed by Maris & Maris (2003). Regarding the power, both the Weibull and the Diffusion

Superposition Model studies showed a similar pattern of results. In particular, the power of the mixture KS test tends to be lower than the power of the truncated KS test as sample size increases.